

# Lorentz invariant entanglement distribution for the space-based quantum network

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(Dated: April 18, 2017)

In recent years there has been a great deal of focus on a globe-spanning quantum network, including linked satellites for applications ranging from quantum key distribution to distributed sensors and clocks. In many of these schemes, relativistic transformations may have deleterious effects on the purity of the distributed entangled pairs. This becomes particularly important for the application of distributed clocks. In this paper, we have developed a Lorentz invariant entanglement distribution protocol that completely removes the effects due to the relative motions of the satellites.

*Introduction* One of the main roadblocks to the widespread utilization of quantum communication such as quantum cryptography is the difficulty of producing long-distance entanglement. Photons are a natural way of generating such entanglement due to their excellent coherence properties and the fact that they are “flying qubits”. However optical fiber quantum communication is limited to distances of approximately  $\sim 100$  km due to photon loss, which make them practical for only for a limited region and not a global scale. Broadly speaking, two approaches have been considered to overcome this challenge – the use of quantum repeaters to cascade entanglement generation for longer distances [1, 2], and free-space schemes [3–5]. While most free-space schemes so far have been ground-to-ground communication, there is now great activity in towards space-based schemes [6–19]. Quantum communication in space is attractive due to the negligible effects of the atmosphere, which is the origin of decoherence effects such as photon loss and dephasing. The space-based protocol allows for the possibility of globe-scale quantum network where the photons can be transmitted at distances of the order of the diameter of the Earth without the need of additional infrastructure such as quantum repeaters.

Arguably the most widespread example of space-based quantum technology that is in use today is the Global Positioning System (GPS), which is based on quadrangulation from satellites loaded with atomic clocks transmitting their time and position. It is well known that relativistic effects due to both special and general relativity must be accounted for an accurate determination of the position, as time dilation and the gravitational red shift affect the clock rate due to the orbital motion of the satellites. In addition it is known that relativistic

effects have an influence upon entanglement [20–22]. For instance in Ref. [20], it was shown that entanglement may change when viewed from different frames for polarization-encoded photon pairs, due to the polarization not being a Lorentz invariant (LI) quantity. Similarly, entanglement encoded in terms of the frequency of the photon are not immune to relativistic effects as energy is not a LI quantity. This suggests that investigating methods of entanglement distribution that have favorable properties in relation to relativity should be an important consideration. One particularly important application where such effects should be important is clock synchronization. Several schemes have been discussed to accurately synchronize atomic clocks on satellites, based on shared entanglement [23–25]. In view of atomic clocks on satellites having a precision of to one part in  $10^{13}$ , and ground-based optical atomic clocks reaching one part in  $10^{18}$  and beyond, even small effects due to relativity become an important consideration. Other potential applications in addition to cryptography and clock synchronization are quantum metrology, quantum distributed computing, quantum teleportation, quantum simulation, and super-dense coding [26–29]. To date, we are aware of no systematic study has been made to investigate such relativistic effects during space-based photonic entanglement distribution, and propose favorable ways of overcoming these issues.

In this paper, we investigate various strategies for space-based entanglement distribution using photons. We examine three popular alternatives for entanglement generation: (I) a polarization entangled photons; (II) single photon entangled state; and (III) dual rail entangled photons. The advantages and disadvantages of each will be investigated in the context of low Earth orbit (LEO)

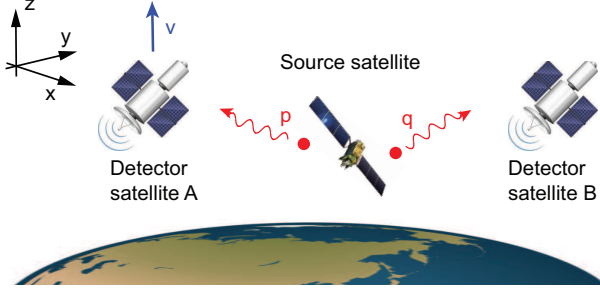


FIG. 1. Entanglement distribution between three satellites in LEO. The source satellite produces entangled photons as shown in the text. The detector satellites are moving with respect to the source satellite and each other. The photons heading to the two satellites may have different momenta  $\mathbf{p}, \mathbf{q}$ , due to their different directions. We choose Alice's satellite to be moving in the  $z$ -direction without loss of generality.

satellites producing and detecting the photons (see Fig. 1). The photonic states (II) and (III) are particularly interesting as they are based on Fock states, which are LI quantities [30]. It is therefore natural to choose entangled states involving these degrees of freedom to develop a truly Lorentz invariant (LI) entanglement distribution. Choosing such manifestly LI states bypasses the need for any correction that would need to be made for states such as (I). We analyze the prospects for whether making such a correction would be viable, and examine to what extent the relativistic effects would be visible.

*Entangled states* Let us first introduce the three types of entangled photon states that will be analyzed in this paper for creating long-distance entanglement using photons. The first is simply a polarization entangled photon pair, produced for example by parametric down conversion. The state is written

$$|\Psi_I^{(S)}\rangle = \frac{1}{\sqrt{2}} (|\mathbf{p}, h\rangle_A |\mathbf{q}, h\rangle_B - |\mathbf{p}, v\rangle_A |\mathbf{q}, v\rangle_B), \quad (1)$$

where  $|\mathbf{p}, \sigma\rangle$  is a single photon state of four momentum  $\mathbf{p}$  and polarization  $\sigma = h, v$ , and the  $S$  refers to the fact that the photons are in the reference frame of the source satellite. We label the modes for Alice and Bob's satellites with  $A$  and  $B$  respectively. The second type of entangled state is the single photon entangled state, which can be produced by a single photon source mounted on the source satellite entering a 50:50 beamsplitter. The state is

$$|\Psi_{II}^{(S)}\rangle = \frac{1}{\sqrt{2}} (|\mathbf{p}, \lambda\rangle_A |0\rangle_B - |0\rangle_A |\mathbf{q}, \lambda\rangle_B). \quad (2)$$

where  $\lambda = \pm 1$  labels the helicity, and  $|0\rangle$  is the electromagnetic vacuum. Finally, the third type of entangled state is using a dual rail encoding, where Alice and Bob each possess two distinct modes  $A1, A2$  and  $B1, B2$  respectively, and the same helicity is used for both photons

and modes:

$$|\Psi_{III}^{(S)}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_{A1} |\mathbf{p}, \lambda\rangle_{A2} |0\rangle_{B1} |\mathbf{q}, \lambda\rangle_{B2} - |\mathbf{p}, \lambda\rangle_{A1} |0\rangle_{A2} |\mathbf{q}, \lambda\rangle_{B1} |0\rangle_{B2}). \quad (3)$$

Each of these states will have a different behavior under a Lorentz transformation, and our task will be to identify which is the best for entanglement generation.

*Lorentz boost of a single photon* First, let us examine how single photon states transform. For a photon of helicity  $\lambda$  and momentum  $\mathbf{p}$  in the Source frame, the state in Alice's frame is

$$U(\Lambda)|\mathbf{p}, \lambda\rangle = e^{-i\lambda\Theta(\Lambda, \mathbf{p})} |\Lambda\mathbf{p}, \lambda\rangle \quad (4)$$

where  $\Theta$  is the Wigner phase, and  $\Lambda$  is the Lorentz transformation to the frame of  $A$ . Since we assume that the photon momentum is in an arbitrary direction, without loss of generality we may take the Lorentz transformation to be a pure boost in the  $z$  direction  $\Lambda = L_z(\beta)$ . In this case  $L_z(\beta)$  is the standard Lorentz transformation matrix with dimensionless velocity  $\beta = v/c$  ( $c$  is the speed of light). Polarized vectors in the original frame are defined as [20]

$$\begin{aligned} |\mathbf{p}, h\rangle &= R(\hat{\mathbf{p}})(0, \cos \phi, -\sin \phi, 0)^T \\ |\mathbf{p}, v\rangle &= R(\hat{\mathbf{p}})(0, \sin \phi, \cos \phi, 0)^T \\ |\mathbf{p}, \lambda\rangle &= R(\hat{\mathbf{p}})(0, 1, i\lambda, 0)^T / \sqrt{2} \end{aligned} \quad (5)$$

where the rotation matrix is  $R(\hat{\mathbf{p}}) = R_z(\phi)R_y(\theta)$ , with  $R_{y,z}$  being the standard  $SO(3)$  rotation matrices, and  $\hat{\mathbf{p}} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$  is the normalized 3-momentum. For a pure boost in the  $z$  direction, the effect is to transform the coordinates as

$$\begin{aligned} \sin \theta &\rightarrow \sin \theta' = \frac{\sin \theta}{\sqrt{\sin^2 \theta + \gamma^2 (\cos \theta - \beta)^2}} \\ \phi &\rightarrow \phi' = \phi. \end{aligned} \quad (6)$$

To a good approximation, for  $\beta \ll 1$  the variation in angle has the effect of

$$\theta' \approx \pi \left( \frac{\theta}{\pi} \right)^{1 - \frac{2}{\pi \ln 2} \beta}. \quad (7)$$

This effectively broadens or contracts the angular variation around the  $z$ -axis. The angular variation is the origin of the variation in entanglement that was observed in works such as Ref. [20].

It is known that polarization is not a LI quantity and hence the state will appear differently in Alice's frame [20, 31]. To quantify the change we measure the trace distance of the polarization vector

$$\varepsilon = \text{Tr}(\sqrt{(\rho^{(S)} - \rho^{(A)})^2})/2 \quad (8)$$

where  $\rho^{(S)} = \text{Tr}_{\mathbf{p}}(|\mathbf{p}, \sigma\rangle\langle\mathbf{p}, \sigma|)$  and  $\rho^{(A)} = \text{Tr}_{\mathbf{p}}(|\Lambda\mathbf{p}, \sigma\rangle\langle\Lambda\mathbf{p}, \sigma|)$  for this case. Here we trace

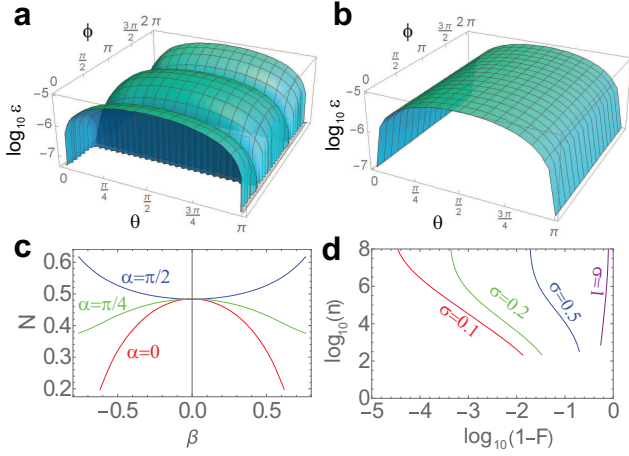


FIG. 2. Performance of the entanglement distribution for various protocols. Trace distance  $\varepsilon$  between the original state and that observed in a moving frame for (a) a single horizontally (or vertically) polarized photon (b) a polarization entangled photon pair moving in opposite directions  $\theta = \theta_A = -\theta_B$ . Parameters are  $\beta = 10^{-5}$ . (c) Negativity of (10) under Lorentz boosts with different orientations. Photons are taken to move in opposite directions  $\theta_A = -\theta_B$ ,  $\phi_A = \phi_B$  and the spread due to the diffraction is  $\sigma = 1$ . (d) Number of entangled photon states (10) with  $\sigma = 1$  required to reach purities as marked. We assume a photon attenuation factor of  $\mathcal{A} = 100$ , and the number of photons required for  $k$  purification steps to be  $2^k$ .

over the momentum degrees of freedom in order to obtain a  $4 \times 4$  matrix that is with respect to the polarization degrees of freedom. Fig. 2(a) shows the trace distance between a horizontally polarized photon with momentum  $\mathbf{p}$  as observed by the source and Alice's satellite. For small velocities  $\beta \ll 1$  as will be true for all satellites orbiting the Earth, expansion of the density matrices reveals that

$$\varepsilon_h \approx \beta \sin \theta \cos \phi, \quad (9)$$

which very accurately summarizes the numerical results in Fig. 2(a). For photons traveling along the  $y$  or  $z$  axis there is no effect as horizontally polarized photons are aligned along the  $x$ -axis. We see that the basic effect of the relativistic correction on the polarization is at the level of  $\varepsilon_h \sim O(\beta)$ . We note that the trace distance is the most appropriate quantity (than the fidelity for instance which scales as  $F \sim 1 - O(\beta^2)$ ), as it is most closely related to distances on the Bloch sphere. For example, in interferometric measurements, the error in the phase is proportional to the trace distance between the ideal and the state with error [23].

*Lorentz boost of entangled states* Let us now examine the effect on the entangled states. For the type I entangled state, in Alice's frame we have

$$|\Psi_I^{(A)}\rangle = \frac{1}{\sqrt{2}} (|\Lambda \mathbf{p}, h\rangle_A |\Lambda \mathbf{q}, h\rangle_B - |\Lambda \mathbf{p}, v\rangle_A |\Lambda \mathbf{q}, v\rangle_B). \quad (10)$$

The Wigner phase does not affect the state in this case as the state is transformed only by a pure Lorentz boost. The sole effect in terms of the trace distance is the rotation of the polarization vectors, as given in (6). The trace distance between the states in the Source and Alice's frames  $\rho^{(S,A)} = \text{Tr}_{\mathbf{p},\mathbf{q}}(|\Psi_I^{(S,A)}\rangle\langle\Psi_I^{(S,A)}|)$  is shown in Fig. 2(b). For the case of photons moving in opposite directions, the trace distance can be summarized to a very good approximation by

$$\varepsilon_I \approx \beta \sin \theta. \quad (11)$$

We again see that the relativistic correction again occurs at the level of  $\sim O(\beta)$ .

For satellites in LEO typically  $\beta \approx 10^{-5}$ , hence this is significant effect in comparison with the precision of atomic clocks. For example, in the clock synchronization scheme of Ref. [23], if Alice and Bob measure in different bases, this appears as an offset in the time between their clocks [32]. One may argue that such systematic errors such as (9) or (11) can always be accounted for, and hence removed. This is indeed true for GPS satellites where relativistic effects such as time dilation are compensated out. In this way the errors could potentially be reduced to a level below (9) or (11). Then the real error estimate is then determined by how well the relativistic corrections can be corrected out, which for the case (9) is related to the error on the velocity estimate  $\delta\beta$ . This gives an error of  $\varepsilon \sim O(\delta\beta)$  for (11). Since the precise velocities of the satellites are typically not known to extremely high precision, the relativistic errors can be significant, even if they are accounted for. For example, if the velocity of the satellite is known with relative error of  $\sim 10^{-6}$  [33], thus amounts to an error  $\varepsilon \sim 10^{-11}$ , which is still large in comparison to the precision of atomic clocks.

In this regard, the type II and III entangled states are a better choice. Fock states, including the vacuum, are known to be invariant states under Lorentz transforms, and remain orthogonal in all reference frames. For the single photon entangled states, transforming to the reference frame of satellite  $A$ , we find

$$|\Psi_{II}^{(A)}\rangle = \frac{1}{\sqrt{2}} (e^{-i\lambda\Theta(\Lambda,\mathbf{p})} |\Lambda \mathbf{p}, \lambda\rangle_A |0\rangle_B - e^{-i\lambda\Theta(\Lambda,\mathbf{q})} |0\rangle_A |\Lambda \mathbf{q}, \lambda\rangle_B). \quad (12)$$

$$|\Psi_{III}^{(A)}\rangle = e^{-i\lambda(\Theta(\Lambda,\mathbf{p})+\Theta(\Lambda,\mathbf{q}))} (|0\rangle_{A1} |\mathbf{p}, \lambda\rangle_{A2} |0\rangle_{B1} |\mathbf{q}, \lambda\rangle_{B2} - |\mathbf{p}, \lambda\rangle_{A1} |0\rangle_{A2} |\mathbf{q}, \lambda\rangle_{B1} |0\rangle_{B2}), \quad (13)$$

Similarly to type I, the photons are Lorentz transformed and there are separate Wigner phase terms due to the photon traveling with different momenta. Helicity is a Lorentz invariant quantity. The Wigner phase, which depends on the Lorentz transformation and the momentum does not show up in the measure we calculate. In both these cases, the entanglement is present in the photon number, rather than polarization. We thus define the density matrices for these states according to

$$\rho = \text{Tr}_{\mathbf{p},\mathbf{q},\lambda}(|\Psi\rangle\langle\Psi|). \quad (14)$$

The trace distance between  $\rho^{(S)}$  and  $\rho^{(A)}$  is always zero, hence it is a manifestly LI state.

*Diffraction effects* Up to this point, we have made one idealization in that the effects of photon diffraction were not included. In a more realistic situation, the photons will have a spread due to diffraction and will have a superposition of different momenta  $\mathbf{p}, \mathbf{q}$ . All three types of states that were considered (10), (12), (13) in fact have the same entanglement as a maximally entangled Bell state in all frames. As discussed in Ref. [20], relativistic effects can affect the amount of entanglement as it changes the diffractive spread of the photons. This type of error is of relevance to our case as it is not a systematic error that is correctable through local operations on Alice and Bob's satellites.

We estimate the magnitude of these corrections for the three types of photonic entangled states. To take into account of diffraction, we integrate with a momentum distribution [20]

$$|\tilde{\Psi}\rangle = \int \tilde{d}\mathbf{p} \tilde{d}\mathbf{q} f_A(\mathbf{p}) f_B(\mathbf{q}) |\Psi(\mathbf{p}, \mathbf{q})\rangle \quad (15)$$

where the  $|\Psi(\mathbf{p}, \mathbf{q})\rangle$  are the states (1), (2), (3) in the source satellite's frame. Here  $\tilde{d}\mathbf{p} \equiv \frac{d^3\mathbf{p}}{2|\mathbf{p}|}$  is a Lorentz-invariant momentum integration measure and the  $f(\mathbf{p})$  is a normalized diffraction function. For a specific model of the photon spread, we follow the same form as that given in Ref. [20] where only angular spread of photons were considered, and the magnitude of the momentum is set to a constant:

$$f(\mathbf{p}) = \frac{1}{\sqrt{M}} e^{-\frac{\theta^2}{2\sigma^2}} \delta(|\mathbf{p}| - p_0). \quad (16)$$

This gives a Gaussian spread for a photon traveling in primarily the  $z$ -direction.  $\sigma$  is a parameter controlling the angular spread of the beam and  $M$  is a suitable normalization factor. To have photons traveling in directions other than the  $z$ -direction, we make rotation of the coordinates around the  $y$ -axis by changing variables in the integrand

$$\begin{aligned} \theta &\rightarrow \theta'' = \cos^{-1}(\cos \alpha \cos \theta + \sin \alpha \sin \theta \cos \phi) \\ \phi &\rightarrow \phi'' = \tan^{-1} \left( \frac{\sin \theta \sin \phi}{\cos \alpha \sin \theta \cos \phi - \sin \alpha \cos \theta} \right) \end{aligned} \quad (17)$$

which gives photons traveling in primarily the direction  $(\theta, \phi) = (\alpha, 0)$ . To transform to Alice's frame, one then applies a boost in the  $z$ -direction to the states, which amounts to making the transformation (6).

Figure 2(c) shows the entanglement as a function of the satellite velocity for type I photons traveling in opposite directions and various boost angles. In contrast to previous works [20], for boosts aligned to the photon propagation ( $\alpha = 0$ ), we find that the entanglement always degrades regardless of direction. This is due to the different geometry that we consider that is relevant for our case. For photons traveling in opposite directions,

the Gaussian distribution tightens for one of the photons but broadens for the other photon according to (7), which always results in a degradation of the entanglement. For boosts that are perpendicular to the photon propagation ( $\alpha = \pi/2$ ), the entanglement can be increased, as the Gaussian spread is redistributed towards the  $z$ -axis, resulting in an effective tightening of the distribution.

We now estimate the order to which the relativistic corrections affect the entanglement. To gauge this we calculate the effect of the boost on the purity of the states  $P = \text{Tr} \rho^2$ . The purity is directly related to the entanglement in this case as for the case with no diffraction, the entanglement is invariant under all boosts. The degradation in the entanglement observed in Fig. 2(c) arises from an effective decoherence entering the system due to tracing out the momentum degrees of freedom. Performing an expansion for  $\beta \ll 1$  we find that the purity behaves as

$$P \approx 1 - 2\sigma^2(1 + |\beta|)^2. \quad (18)$$

As expected for no diffraction  $\sigma = 0$ , there are no relativistic corrections. The relativistic corrections to lowest order act to accentuate the diffraction effects which are already present. In terms of physical parameters, the diffraction angle can be estimated as  $\sigma \approx \lambda/d$ , where  $\lambda$  is the photon wavelength and  $d$  is the diameter of the transmitter. For infrared photons, this gives  $\sigma \sim 10^{-6}$ . We see that in this case the relativistic corrections are quite small as it is a secondary correction.

Diffraction effects can be remedied using entanglement purification methods. We demonstrate that it is possible to achieve high purities by adapting the purification procedure devised in Ref. [34] to our relativistic entangled photons containing three components (a photon has spin-1). The procedure is similar to original protocol except that due to the additional components one obtains a multivariable recurrence relation instead of a single variable recurrence relation [35]. In Fig. 2(d) we show the results of the entanglement purification on the state (14) using (15). We calculate the number of photons required as the number of photons required for a purification of a particular target fidelity, multiplied by the photon attenuation factor (the ratio of the number of photons sent to received), divided by the success probability of the purification. The photon attenuation is  $\mathcal{A} = L^2 \lambda^2 / d_S^2 d_A^2$ , which for parameters  $L = 13000$  km,  $\lambda = 800$  nm,  $d_S = d_A = 1$  m gives  $\mathcal{A} \approx 100$  photons being sent for each one received [36]. For the various diffractive spreads  $\sigma$  considered, we find that an improvement in the fidelity is achievable as long as the original diffractive spread is lower than  $\sigma \lesssim 1$ . For very broad  $\sigma$  the purification fails and the fidelity decreases. As typically the spread is  $\sigma \ll 1$  we anticipate that such purification methods should always be successful in practice.

Turning to type II and III states, we find that the effects of diffraction that afflicted type I states are not present in terms of entanglement degradation. The reason is that the type of entanglement is encoded in the

orthogonality of the Fock states, which are preserved as they are LI. For the type II state the main effect that one must account for is simply photon loss, which is captured by the photon attenuation  $\mathcal{A}$  which is the same as the above. Various methods exist to perform measurements that are in a superposition basis of the vacuum and a single photon [37–39]. For the dual rail type III states, there is however the issue that the diffraction cone for the two rails will start to overlap unless they are separated by a sufficiently large distance, which is impractical for satellite based sources and detectors. In this case, time-bin entangled modes are a better alternative, with a Franson interferometer performing the interference between the two modes [40, 41]. Relativistic effects will time dilate the time bins (in the same way a spatial separated dual rail will undergo Lorentz contraction) but these are on a much longer timescale, hence should not impact the performance the detection scheme.

**Conclusions** In summary, we have analyzed several photon-based entanglement distribution protocols for the space-based quantum network. We find that standard polarization-based photon entanglement (type I) can experience significant errors for satellites that are in LEO. While in principle these are correctable if the velocities of the satellites are known to high precision, this can still introduce errors at the  $\delta\beta$ , which is the error on the estimate of the satellite velocity. We note that other types of encodings, such as in energy or time, would also undergo Lorentz transformations. Combined with the fact that diffraction effects degrade the entanglement for type I states, our results point to the fact that single photon entangled states (type II) and dual rail photon entangle-

ment (type III) are a superior choice in terms of robustness to relativistic transformations.

One of the primary applications of space-based entanglement is clock synchronization, which is currently performed using classical signals, which requires precise knowledge of the position of the satellites. Entanglement-based methods can potentially eliminate this requirement, but as have shown in this paper, to properly take advantage of this manifestly Lorentz invariant states should be used. Encoding in the Fock state basis, using either a single photon or dual rail encoding overcomes this issue. In addition, the entanglement can be used for several important tasks such as quantum cryptography which can be used without further components such as a quantum memory. For applications that require a quantum memory to further manipulate the entanglement, it is likely necessary to have in addition a LI entanglement transfer and storage, if one requires a high fidelity protocol.

T. B. would like to acknowledge support from the Shanghai Research Challenge Fund, New York University Global Seed Grants for Collaborative Research, NYU-ECNU Institute of Physics at NYU Shanghai, National Natural Science Foundation of China (Grant No. 61571301), the Thousand Talents Program for Distinguished Young Scholars (Grant No. D1210036A), and the NSFC Research Fund for International Young Scientists (Grant No. 11650110425). J. P. D. would like to acknowledge support from the US Air Force Office of Scientific Research, the Army Research Office, the National Science Foundation, and the Northrop-Grumman Corporation.

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